BVR's Guide to Discounts for Lack of Marketability

Volume 1

Fifth Edition

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John Stockdale Sr.

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What It's Worth

BVR's Guide to Discounts for Lack of Marketability, Fifth Edition

VOLUME ONE

By John J. Stockdale Sr., ASA, CPA/ABV

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What It's Worth
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What It's Worth

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Introduction

Dear Reader,

I have been valuing businesses for a long time, and, throughout my career, discounts for lack of marketability (DLOM) have been a hot issue for the business valuation profession. Although it seems that some consensus about the best methods or a generally accepted procedure for DLOM should have been reached by now, I do not believe it has. To the contrary, there is much more diversity in opinion now than there was 35 years ago. Beliefs about the size of the discount, the available evidence, and the computational methods have all broadened—and the trend continues. For example, since I wrote the last edition of this book in 2011, the number of restricted stock studies expanded from 20 to 32 and the number of studies providing analysis of factors doubled from 13 to 26.

When I began focusing on DLOM in the 1970s, the hot new technology to determine the size of DLOM was using averages from restricted stock studies showing the percentage of average discounts to be in the low 30s. At first this seemed to settle the DLOM debate, but ongoing scrutiny raised doubts about the accuracy and wisdom of using only an average to reach a conclusion. It took a few years, but we've certainly moved well beyond the application of these averages.

It is to be expected that the level of detail involved in a DLOM analysis has increased because it is an area in which a percentage—sometimes a large percentage—is cut off at the top of the value of or interest in a business. This certainly warrants attention, and it is no wonder, then, that much more detail goes into the analysis these days. This attention has resulted in the development of different forms of evidence and techniques to apply to the evidence to reach a DLOM conclusion.

Although we have made strides in developing computational methods—some that are very complex—one thing remains clear: No method removes the need for judgment in selecting and interpreting appropriate evidence and computational technique. The currently available computational methods do not conclusively explain the variation shown in the evidence. Human intelligence and judgment remains necessary to bring order to the process of sorting out the meaning of the evidence and methods. This is just as true for the determination of DLOM as it is for the entire field of business valuation.

Note that this book concerns the concept of DLOM as it applies to business valuation only—the valuation of businesses, ownership interests, and securities. It is not intended to cover so-called blockage discounts, which apply to publicly held securities.

The concept of DLOM could theoretically apply to other appraisal disciplines such as valuation of machinery and equipment or real estate. However, professionals in these disciplines have their own ways of handling this issue. Accordingly, this book is intended to be specifically applicable to business valuation.

This book provides a summary of relevant theory, evidence, and techniques useful in reaching a conclusion for the appropriate size of a DLOM. By assembling this material, it is my intent to take the first steps in creating a body of knowledge that eventually results in broad and generally accepted principles for DLOM determination as well as a more consistent practice. Here, I include all forms of credible theory, evidence, and computational methods currently available. Chapter 1 presents the theory. Chapters 2 through 7 provide the various forms of evidence. Chapters 8 through 16 provide the different techniques that have been developed to determine the amount of a DLOM. Chapter 17 summarizes my view of the current state of DLOM affairs.

Sincerely,

John J. Stockdale Sr., ASA, CPA/ABV

To my wife Sylvia, and to our three children, John, Ruth, and Christian.
You all add a great big plus to my life. Thanks for the memories!

About the Author

John J. Stockdale Sr., ASA, CPA/ABV, is a sole practitioner located in the Detroit area. He has 34 years of experience and a broad-based practice that includes valuations for federal tax, legal and divorce, and ESOP purposes, as well as transactions. He has written extensively about business valuation matters, including several articles regarding DLOMs and the current and previous editions of *BVR's Guide to Discounts for Lack of Marketability.* He is a former chairman of the ASA Business Valuation Standards Subcommittee and former chairman of the ASA Business Valuation Committee. He has also testified as an expert witness in numerous matters related to valuation and associated accounting matters.

Chapter 1.
Fundamentals of the Discount for Lack of Marketability

1.1 History of the DLOM

The concept of a discount for lack of marketability was born in the federal income tax area but is now an important consideration in other areas of the law. The first mention of it appears to be in a federal tax case in 1934. From then until the early 1970s, the amount of discount in tax cases increased over time and eventually hovered in an average range of 20% to 30%. The evidence presented in these cases principally consisted of unsupported opinion evidence. Then, in the early 1970s, restricted stock evidence was presented and the discount increased to an average of about 34%.[1] The average discount has not increased much in the vast majority of court cases since then. In fact, it may even have declined because no one has been able to present better evidence or a more compelling computational method. The 34% average turns out to be based on data from a specific period of time and for specific purchasers. Also, it has now become clear that the available evidence shows that the discounts in the studies vary from transaction to transaction. As a result, it appears logical that the DLOM for a specific circumstance should vary depending on the specific facts.

It's particularly tricky to determine a DLOM for an illiquid security, especially if the security represents a minority interest in a closely held firm. That's because there is very little direct evidence that supports either a value or a discount for that specific type of security. The calculation is done by using methods to determine the value of publicly traded securities, with adjustments to account for the additional risk and illiquidity inherent in the securities of a private firm. Differences between the public and private firms are recognized in the rate of return and through the application of a discount for lack of marketability. In the DLOM, the illiquidity is recognized by various methods that consider such factors as the time to sell, unusual costs to market, and an additional liquidity premium in the rate of return. In the case of an illiquid security in a public firm, a discount must also be determined to account for the additional illiquidity.

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[1] Thomas W. Dant Jr., "Courts Increasing Amount of Discount for a Minority Interest in a Business," *The Journal of Taxation*, August 1975.

Methods used to determine the discount for lack of marketability have evolved substantially over time. Initially, the proof was an unsupported assertion, based on experience, that an illiquid security is worth less than a liquid one. Then, in the early 1970s, starting with the SEC, a number of evidentiary studies were developed that compared the price of a security publicly trading on a public market with an identical but restricted security, and these are called "restricted stock studies." A different sort of study followed that compared the price paid for a security while a company was private to the price obtained for a similar security when the company went public in an IPO a short time later. These have come to be called "pre-IPO studies." In the last decade, additional sources of evidence have been developed, but the restricted stock and pre-IPO studies continue to be the most relied-upon evidence.

Initially, the accepted practice was to use averages from the restricted stock studies as an indication of the discount, and, later, averages from pre-IPO studies were added. In the 1990s, there was a growing discontent with the use of averages, particularly by tax court judges, and the idea of using various factors to judgmentally support a discount within the range of the studies came to the fore. The evidence available today certainly bears out the discontent. As will be discussed in Chapter 2, the 34% average is based on data specific to the late 1960s and early 1970s and specific to certain mutual funds. Also during the 1990s, computational methods and theoretical models began to be developed that can be used to compute a discount. A further development has been the availability of commercial databases that provide a practitioner with access to the actual restricted stock and pre-IPO transactions. Although these advances are moving practice forward, no consensus has yet been reached on the best way to determine the discount, and it seems likely that there will be further developments in the field. A variety of available evidence and computational methods will be discussed in this book.

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Initially, the accepted practice was to use averages from these studies as an indication of the discount. In the 1990s, there was a growing discontent with the use of averages, particularly by tax court judges, and the idea of using various factors to judgmentally support a discount within the range of the studies came to the fore. About this time, economic models began to be developed. These computational methods or theoretical models can be used to compute a discount. A further development has been the availability of commercial databases that provide a practitioner with access to the actual restricted stock or pre-IPO transactions. Despite all these advances, no consensus has yet been reached on the best practice to determine the discount, and it seems likely that there will be further developments in the field. A variety of available evidence and computational methods will be discussed in this book.

1.2 The Need for a Discount for Lack of Marketability

The concept of discount for lack of marketability has long been in existence. Why is there such a concept? The need for a DLOM stems historically from the definition of value used in the legal system, primarily the federal tax system, combined with the economic truth of the effects of illiquidity. The definition of value used for taxes is fair market value: "the price at which the property would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or sell and both having reasonable knowledge of relevant facts." (*Estate Tax Regulations* 20.2031-1 (b))

Practitioners generally accept that the term "price" means a price in terms of cash. Any other definition would not make sense. For example, an early and continuing use for the fair market value definition is in the taxation of estates and gifts. The purpose of this tax is to collect cash. Therefore, the basis of a value in cash is a consistent measure to use for the collection of taxes in cash. This concept has been explicitly stated in the definition of fair market value used in the *International Glossary of Business Valuation Terms:* "the price, *expressed in terms of cash equivalents*, at which an asset would change hands between a hypothetical willing and able buyer and a hypothetical willing and able seller, acting at arm's length in an open and unrestricted market, when neither is acting under compulsion to buy or sell and when both have reasonable knowledge of the relevant facts." One tax court memorandum decision (*Estate of Star C. Simpson,* T.C. Memo 1994-207 (May 11, 1994)) explicitly includes the word "cash" in its definition of fair market value. It states the definition of fair market value to be: "The value of property is the price in cash at which such property would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or to sell, and both having reasonable knowledge of all relevant facts."

The phrase "in terms of cash equivalents" is a key element giving rise to the need for a DLOM. The application of a valuation procedure provides an indication of value, but that indication is not necessarily the price that can be quickly obtained in cash. Rather, the valuation procedure provides an indication of the value that will be obtained when a sale eventually occurs. This gives rise to another key concept, which is the time of receipt. Fair market value contemplates a price that is not only in cash, but also in cash that can be received in a relatively short time.

The requirement to determine the value in cash is a sensible one. However, the economic information available and the valuation theories in common use do not result in a value in cash at the valuation date. The application of these theories and data arrives at an investment value that may be the market price eventually achieved or value representative of the cash value. A link is needed between the requirement for a cash price and the value resulting from the economic data and theories.

If there is a different definition of value in which price and time of receipt are not key issues, a discount for lack of marketability is not needed. For example, in the legal area of divorce, in some states, the standard of value is the value to the current holder continuing in the holder's hands and not a value in cash assuming a sale in the near term. In addition, the legal fair value in shareholder disputes in some states may not assume an immediate sale in terms of cash. In such cases, the DLOM is not relevant.

In addition, the concept of DLOM isn't relevant when the objective is to determine the eventual selling or purchase price based on a value that will be realized, even if it will take a period of time consisting of marketing and closing activities to achieve the value. This value may not be immediately received in cash, but may, instead, be received over time or with something equivalent, such as stock or a note paid over time.

1.3 Requirements of Valuation Standards

The concept of DLOM not only makes legal and economic sense, but it is also generally accepted by appraisal professionals when it is applicable. The Uniform Standards of Professional Appraisal Practice and each of the four U.S. organizations that qualify business appraisers, as well as the Canadian Institute of Chartered Business Valuators, recognize the concept of marketability in their standards. Each of these five organizations has adopted the *International Glossary of Business Valuation Terms,* which includes definitions for discount for lack of marketability, marketability, and liquidity.

USPAP Standards Rule 9-4 (d) provides that an appraiser must analyze the effect on value, if any, of the extent to which the interest appraised is marketable and/or liquid.

The comment to this section provides that an appraiser must analyze factors such as holding period, interim benefits, and the difficulty and cost of marketing the subject interest. Standards Rule 10-2 (a) (v) provides that the appraiser must report on the extent to which the interest appraised lacks marketability and/or liquidity.

The AICPA in its Statement on Standards for Valuation Services SSVS-1, Paragraph 40, provides that an appraiser should consider an adjustment for a discount for lack of marketability. Paragraph 63 provides that the valuation report should discuss the valuation adjustment, the rationale for using it, the factors considered in selecting the amount or percentage used, and the preadjustment value to which the adjustment was applied.

The National Association of Certified Valuation Analysts provides in its Professional Standards, Paragraph 3.12, that marketability and liquidity must be considered in a valuation.

The Institute of Business Appraisers, in its Business Appraisal Standards (Publication P-311c Paragraph 7.5 (e) (v) & (vi)), provides that the professional shall consider the degree to which the subject business and the subject interest is marketable compared to its peer group.

The American Society of Appraisers provides ASA Business Valuation Standard BVS-VII concerning valuation discounts and premiums. Among other issues, this standard provides that the base value to which a discount is applied must be defined and that a discount quantifies the adjustment to account for differences in the base value and the value of the subject.

1.4 Theory of the DLOM

1.4.1 Definition of the DLOM

A correct determination of a DLOM must be based on the proper definition of the term. It is key at the outset to use an accepted definition of DLOM as a basis for developing methods to quantify it. Fortunately, there is general acceptance of such a definition. All of the organizations mentioned in the previous section accept the *International Glossary of Business Valuation Terms*. The glossary defines discount for lack of marketability (DLOM) as “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.” It defines marketability as “the ability to quickly convert property to cash at a minimal cost.” Combining the two definitions allows us to arrive at a more detailed definition of DLOM: “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of the ability to quickly convert property to cash at a minimal cost." Thus, by definition, the DLOM is a reduction in value resulting from an inability to sell the property quickly at a minimal cost. This definition incorporates the key points that have developed in the legal framework.

Liquidity is a concept that is integrally related to marketability. The *International Glossary* also has a definition for liquidity. Liquidity is "the ability to quickly convert property to cash or pay a liability." A comparison of this important term shows that liquidity is included under the umbrella term of "marketability." Both imply the ability to quickly convert property to cash, but marketability also includes the concept of "at a minimal cost." A security could have a DLOM resulting from the fact that it is illiquid. It could also have an increase in a DLOM because it cannot be sold at low cost. Both parts of this definition are important because they are elements buyers and sellers will likely consider.

It is important to note, however, that nothing in the term "marketability" limits it to applying only to liquidity. All factors that would account for a reduction in value from an immediate cash price should be considered. It is the connection to the cash price that is the subject of the definition, not the nature of the factor causing the reduction. Thus, factors such as information uncertainty, monitoring, or compensation are within the purview of the definition to the extent that they reduce price.

Some analysts use different definitions for both marketability and liquidity. They would define marketability as the right to sell a security and liquidity as the ability to sell a security. The *International Glossary* does not recognize this distinction and never uses the term "right." The term "ability," as used in the above definitions, is assumed to incorporate the right to sell. One would not have the ability to sell unless the right to sell was incorporated within it.

Nothing in the definition of DLOM seems to limit its applicability to any particular type of security. If the security cannot be sold for a cash price in a reasonably short period, a discount for lack of marketability should be applied. For example, let's say a partnership owns an investment in bank CDs and a limited partner does not have the ability to obtain value immediately. The concept of DLOM seems to apply to the partnership interest because the partner and any potential buyer cannot obtain the value of the interest in a reasonably short time. The logic of this can be shown by asking the question: "Which would you rather have: cash in hand or an interest in a partnership in which you are assured of getting the same cash value at some future time?"

Conceptually, the idea of DLOM applies equally to control ownership interests and non-control ownership interests. If the holder of a control ownership interest cannot obtain a price in cash in a reasonably short time, a discount for lack of marketability seems to be applicable. The logic of this can be shown—again—by asking the question: "Which would you rather have: cash in hand or a business for which you expect to obtain the same cash value at a future time?" The argument is sometimes advanced because if a controlling ownership can make the decision to sell and the decision to distribute from the business, no discount for lack of marketability should be applied. However, the decision to attempt to sell does not automatically result in the achievement of a cash price in a reasonably short period of time. The issue of DLOMs applied to controlling interests will be discussed in greater detail in Chapter 16.

The DLOM is a link between the indication of value resulting from the application of a valuation approach, such as income, market, or asset-based, and the price stated in terms of cash received in a relatively short period of time.

1.4.2 Mathematical statement of the DLOM definition

The definition of DLOM may be modeled by the following expressions:

$$\text{(1)} \qquad \textit{DLOM} = \textit{Reduction in value due to inability to sell quickly}$$

$$\text{(2)} \qquad \textit{DLOM\%} = \frac{\textit{Reduction in value due to inability to sell quickly}}{\textit{Base value}}$$

$$\text{(3)} \qquad \textit{DLOM} = \textit{Base value - cash value as if sold quickly}$$

$$\text{(4)} \qquad \textit{DLOM\%} = \frac{\textit{Base value - cash value as if sold quickly}}{\textit{Base value}}$$

Where:

> The reduction in value from the inability to sell quickly is the result of the application of the DLOM and is an initial unknown in a typical valuation problem.
>
> Base value = concluded value of the ownership interest resulting from the application of a valuation approach, such as income, market, or asset-based.
>
> Cash value as if sold quickly = current cash price as if sold quickly.
> = present value of cash eventually received from a sale of the interest.

The cash value as if sold quickly will be referred to as the "less liquid value" throughout this book when referring to values in evidence studies.

All of the evidence and models concerning the DLOM are rooted in these mathematical models. Some evidence studies and computational methods are based on Formulas 1 and 2. In these situations, the reduction in value is measured directly. For example, a put option may be used to determine the amount of the reduction. An example of this type of evidence study is a put LEAP, which will be discussed in detail later.

Other evidence studies and computational methods are based on Formulas 3 and 4. An example of an evidence study of this type is a restricted stock study. An example of a computational method is the quantitative marketability discount model in which the base value and current cash price are separately determined.

1.4.3 The concept of base value

In the performance of a valuation, the base value is that determined for the interest before the application of a DLOM. This is the concluded value resulting from the application of valuation approaches, such as income, market, or asset-based. It is the value likely to be obtained in the relevant market if the asset could be sold quickly. The value eventually achieved when the sale is completed may be more or less due to growth and risk factors.

The base value may be the value as if freely trading on an exchange as a minority block, but it is not necessarily that. Many companies that are the subject of a valuation are unlikely to ever have the characteristics necessary to be a public company. The value for these companies is to be found in the relevant market, based on factors of that market, and these values may be fundamentally different from public-market, freely trading values. For example, a small single-location convenience store with $500,000 of revenue will most likely never become a public company, but there is a market for such companies. The base value for such a company is the value in its relevant market.

In an evidence study, the base value is the indicated value from which the reduction is measured. An example is the freely traded market price in a restricted stock study. Another example is the pro rata private equity interest in the Pepperdine Private Capital Markets Project Surveys.

The cash value as if sold quickly is the result of the application of the DLOM to the base value. The definition of the term "quickly" must be carefully considered, however, because gift and estate tax regulations make it clear that fair market value cannot be the result of a forced liquidation.[2] The gold standard for measuring cash receipt for this value can be seen in the price of small minority blocks trading on an exchange when the cash is received in three days from initiating the sale. For most interests, a measurement in this three-day period would result in a truly low value because a number of steps would be required to get the cash in hand. From the decision to sell, this would involve activities such as determining a price, a marketing process for the interest, due diligence on the part of the purchaser, reaching agreement on price, and closing. To induce a purchaser to complete these steps in a three-day period would involve a virtual giveaway price, equivalent to a forced liquidation, which is not what is contemplated. A more reasonable definition of the term "quickly" contemplates a cash value resulting from a hypothetical deliberative sales process with that value reduced to the present value as of the valuation date.

1.5 Base Value and the Relevant Market

The base value is absolutely critical to the application of a DLOM. When applying a discount, the resulting answer is either ambiguous or wrong if the discount and the base are not consistent. The American Society of Appraisers states, regarding this subject: "A discount has no meaning until the conceptual basis underlying the base value to which it is applied is defined."[3] The base value is also important in interpreting and applying the DLOM evidence studies. To reach a correct conclusion of value immediately achieved in cash, the DLOM applied must be consistent with base value determined or these variables must be adjusted to make them consistent.

1.5.1 Levels of value

The initially determined base value could be one of several different conceptual levels of value, which have been described in so-called levels of value charts, and different levels of value charts have been advanced by several practitioners.[4,5,6,7,8] Thinking on this topic has evolved over the years. There now seems to be a consensus that principal levels of value take the form of a chart something like Exhibit 1.

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[2] Treasury Regulations Sec. 20.2031-1 (4) (b) states: "The fair market value of a particular item of property includible in the decedent's gross estate is not to be determined by a forced sale price."

[3] ASA Business Valuation Standards BVS-VII.II.A.

[4] Eric W. Nath, "Control Premiums and Minority Interest Discounts in Private Companies," *Business Valuation Review,* Vol. 9, No. 2, June 1990. 39-46.

[5] Z. Christopher Mercer, "Do Public Company (Minority) Transactions Yield Controlling Interest of Minority Interest Pricing Data?" *Business Valuation Review,* Vol. 9, No. 4, December 1990. 123-126.

[6] Michael J. Bolotsky, "Adjustments for Differences in Ownership Rights, Liquidity, Information Access, and Information Reliability: An Assessment of 'Prevailing Wisdom' Versus the 'Nath Hypothesis,'" *Business Valuation Review,* Vol. 10, No. 3, September 1991. 94-110.

[7] David W. Simpson, "Minority Interests and Marketability Discounts: A Perspective," *Business Valuation Review,* Vol. 10, No. 2, June 1991. 47-50.

[8] Z. Christopher Mercer and Travis W. Harms, *Business Valuation: An Integrated Theory,* 2nd edition. John Wiley & Sons Inc., 2008, 61-93.

Exhibit 1. Principal Levels of Value

Strategic value
Control value
Marketable minority value
Nonmarketable minority value

There is some debate about exactly what levels should be included in Exhibit 1, particularly moving up from the marketable minority value. The general idea of the levels of value chart is that, on a share-for-share basis in the same company, as one moves from a higher level to a lower level in the chart, the value goes from a higher price to a lower price. Both control issues and marketability issues cause the movement from the higher level and value to the lower level and value. Because these issues sometimes go together, it can be difficult to separate them. There is substantial evidence about marketability issues, which will be discussed extensively in this book, but less evidence about control issues.

Marketable minority value is also sometimes known as freely traded value or publicly traded minority value. The traditional view is that there is a base value, which is the marketable minority value. A DLOM is applied to this base to get the nonmarketable minority value, and a control premium is applied to get a control value or a strategic value. Another common view is that the base value is the strategic or control value to which discounts for lack of control and lack of marketability are applied to get the nonmarketable minority value.

1.5.2 Relevant markets

The traditional levels of value chart and the traditional view continue to be useful. However, the levels of value chart has a limitation because it is centered on values in the public markets. Freely traded value is the hub of this chart. However, values in different markets can result from fundamentally different inputs, meaning the levels of value are not always relevant. For example, some private companies are unlikely ever to become public companies, and reference to a freely traded value is strained, at best. A more effective way to use the concept of base value with respect to DLOM to highlight the issue of marketability is to relate each base value to the market in which it lies. Assumptions regarding this market are used in determining the base value in a valuation. In each evidence study, the base value is drawn from a market. The less liquid value is normally drawn from another segment of that market or from another related market with different marketability characteristics. The concept of the particular market becomes highly relevant, and understanding the characteristics of the particular market is important. In fact, the concept of market is so important that the law in federal estate tax matters requires that the sale price be drawn from the market in which it is most commonly sold.[9]

Freely traded value should not always be the hub of a valuation and should not always be the base. Several different types of base values can be determined according to the appropriate market. The appropriate market can be defined along three dimensions:

(1) The percentage ownership interest;

(2) The size of the company in the market; and

(3) Whether the companies are closely held companies and whether they are, or could become, publicly held.

Using these dimensions, several relevant markets for securities can be defined. The markets can be further subdivided along these dimensions. The basic subdivisions are listed below by degree of organization, showing major subdivisions of each market.

1. Minority interests in public companies
    a. Block size continuum affecting liquidity;
    b. Company size continuum affecting liquidity;
    c. Legal characteristics affecting liquidity (degree of restriction);
    d. Company and trading characteristics affecting the specific market on which the company would trade (NYSE, NYSE MKT, NASDAQ, OTCBB, Pink Sheets); and
    e. Secondary markets, such as the American Partnership Board and the Second Market.

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[9] Treasury Regulations Sec. 20.2031-1 (4) (b) states: "Nor is the fair market value of an item of property to be determined by the sale price of the item in a market other than that in which such item is most commonly sold to the public."

2. Control interests that can direct the sale of an entire company
    a. Large companies;
    b. Middle-market companies; and
    c. Small companies.

(This category may also be subdivided into public and private companies, as will become obvious in a later section on paired company studies.)

3. Minority interests in closely held companies
    a. Companies that could become public;
    b. Interests in professionally managed private equity partnerships or of interest to private equity investors; and
    c. Companies that are not likely candidates to become public on the valuation date and are unlikely ever to become public.

Each subdivision could be further broken down into varying degrees of detail.

1.5.3 Characteristics of the markets

Each division and subdivision of the market has its own degree of organization, which results in a different time to sale, different risk of achieving the base value, and different amount of available information. These factors result in a different degree of illiquidity and marketability and will affect the DLOM that should be applied. It is interesting to compare the different characteristics of each market. These factors prove useful when reviewing evidence studies and considering computational methods.

The market for minority interests for public companies is highly organized. There is substantial information about the companies themselves, the prices historically achieved and currently available, and the trading characteristics in the market. The prices reported would generally represent an active marketplace with many buyers and sellers. There is also information about sales of block sizes larger than that normally traded daily. In this market, small block sizes are virtually guaranteed of being sold. Also, information and techniques are relatively advanced to estimate the timing and risk of completing sales of larger blocks. These factors offer substantial liquidity for block sizes within the range normally traded, but also offer increasing illiquidity as the block size increases. Eventually, there is substantial illiquidity as SEC restrictions kick in and the ability of the market to absorb the block in a reasonable time is exceeded.

In the market for whole companies, an organized market exists with investment bankers or business brokers specializing in almost every corner of the market. There is a possibility or, for some companies, a probability of sale, and there is information available about the expected time it will take to sell the company. Substantial information is available about the prices historically achieved as well as related information through various databases and news services. The reported prices would generally represent the highest price achieved in a selling process involving a relatively small number of buyers. For public companies, there is substantial information about the companies that have sold, but for private companies, there is very little. These factors offer some liquidity, but less than in the market for minority interests in public companies.

In the market for minority interests in private companies, there has been no organized market. Securities laws work to prevent the development of such a market and limit the number of buyers who can participate in it. However, the early development of so-called secondary markets is beginning. Sales are completed in this market but mostly from the company to investors in the company. There appear to be few sales among buyers and sellers unrelated to the company. A very limited amount of information is available about actual transactions in this market, including only one older article reporting actual sales of interests and some information from the private equity and venture capital markets. There is also some information about interests in companies that have become public. All in all, these conditions result in very substantial illiquidity in this market.

In the process of valuing a company in any of these markets, the base value may be an indication of value from within the appropriate market. The correct DLOM to apply is the one that links the base value from one of the markets to the less liquid value in the market appropriate to the interest being valued. It is important that both the market in which the base value is derived and the market in which the subject is to be valued be crisply defined. Only then can all of the appropriate elements be included in the DLOM.

The issue that makes the determination and application of DLOMs difficult is that there is a diminishing amount of information available about the size of DLOMs as the degree of organization in the market drops off. At the same time, most of the assignments involve valuations in Markets 2 and 3, where the availability of information is the least.

1.6 Liquidity Concepts

Liquidity is a principal factor affecting marketability, but not the only factor. Nevertheless, the relative liquidity of a security profoundly affects the discount for lack of marketability. A significant volume of academic study has been generated on this topic in the last few years. The basics of liquidity theory are helpful in understanding DLOM evidence studies and computational methods. They are also useful in extending DLOM evidence and methods from information-rich markets to those with lesser information.

Amihud, Mendelson, and Pedersen[10] provide a survey of more than 200 papers on the subject of liquidity theory, which is the source for the following discussion. The discussion lays out four theoretical propositions concerning the relationship between rate of return and illiquidity. A number of assumptions and substantial mathematical development are needed in the understanding of these propositions. The following is a basic statement of the four propositions as well as an explanation of the implication of each proposition for the development of marketability discounts.

1.6.1 Summary of liquidity propositions

These propositions may be summarized as follows:

1. The rate of return increases as the illiquidity of a security increases, essentially a liquidity premium. The greater the illiquidity of the security, the greater the premium.

2. The liquidity premium is a concave function. It increases at a decreasing rate. Thus, discounts for lack of marketability should be greater for less liquid securities, but they will not increase in a linear manner.

3. The liquidity premium will vary over time and will vary differently for different securities. Securities that become more illiquid when the markets become more illiquid will have a higher liquidity premium and, thus, a higher discount for lack of marketability.

4. Liquidity is persistent. A high illiquidity today predicts a high illiquidity in the future. This implies a high required return, which is achieved by lowering current prices.

Empirical studies have shown that these liquidity effects exist, but are difficult to measure. The size of the liquidity premium is of some substantial doubt.

Proposition 1 is the basic model of liquidity and asset prices. This proposition is that the price of any security is the expected present value of all future dividends minus the expected present value of all future transaction costs, taking into account the expected trading frequency. In other words, the price of a security is not only dependent on its dividends, but also on all future transaction costs. Thus, the greater the transaction costs and the more frequent the trading frequency, the greater the illiquidity effect—and the lesser the price of the security and the higher the expected rate of return.

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[10] Yakov Amihud, Haim Mendelson, and Lasse Heje Pederson, "Liquidity and Asset Prices," Pedersen, *Foundations and Trends in Finance*, Vol. 1, No. 4, 2006. 269-364. Available on the Internet via search engine.

Mathematically, this may be stated as follows:

$$E(r_i) = r_f + \mu C_i / P_i$$

Where:

$E(r_i) = D_i / P_i$ = expected rate of return;

D_i = dividends per period;

P_i = price of a security i;

r_f = the risk-free rate;

μ = the trading intensity; and

C_i = the cost per trade.

This proposition means that the expected rate of return of any security incorporates the expected liquidity for that security. For example, the expected rate of return for a small publicly traded company trading on NASDAQ would be higher than the expected rate of return for small blocks of common stock of a large publicly traded company trading on NYSE because the latter shares are more liquid. This would be the result of fewer trades, but higher cost per trade, as reflected in the higher proportional bid-ask spread for the small company compared to the larger company. The expected rate of return on a privately held company would be higher still because these shares are even more illiquid. To trade these shares, an investor would incur substantial costs, such as legal and accounting, and higher broker fees. Under this proposition, rates of return are higher for privately held companies, not only because they are riskier, but also because they are more illiquid. The proper rate of return for any security incorporates the premium associated with the illiquidity of that security. For example, respondents to a survey report that their required rate of return on new investments for private equity is 25% to 30%.[11] Liquidity Proposition 1 indicates that this range of rates of return incorporates liquidity as well as risk.

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[11] Pepperdine Private Capital Markets Project Survey Report, Volume II, Winter/Spring 2010, 63. This is a periodic survey sponsored by Pepperdine University under the supervision of John Paglia, ASA, and first published in 2009. It surveys banks, asset-based lenders, mezzanine investors, venture capitalists, and private equity investors. It provides a broad array of useful information about the opinions of these investors concerning matters of interest to business valuators. It is freely available on the Internet.

It is well accepted that the rate of return increases as the degree of risk increases. This proposition indicates that the rate of return will also increase as the degree of illiquidity increases. Thus, the price of every asset will be based on a rate of return that includes an increment of return resulting from the liquidity premium. This illiquidity is part and parcel of observed rates of return in the market. Every asset that is valued will include some degree of illiquidity because it is based on a rate of return that includes a liquidity premium. This proposition indicates that there is an additional regime in the spectrum of rates of return. There is an increasing liquidity premium in addition to the increasing risk premium. It follows that the base value to which a DLOM is to be applied will include some degree of illiquidity.

Proposition 2 is called the "clientele effect." It adds the concept of economic rent to the basic model stated in Proposition 1. A premise of this proposition is that securities with higher transaction costs are allocated to investors with longer investment horizons. Because these investors trade less often, they will incur fewer transaction costs over the long term. They are less disadvantaged from investing in less liquid securities and can take advantage of the higher returns available in these securities, which are shunned by investors with shorter trading horizons. As a result of this proposition, it can be shown that the expected return is an increasing and concave function of the relative transaction cost C_i/P_i. A concave function continually increases, but at a lesser rate as illiquidity increases.

Mathematically this proposition is stated as follows:

$$E(r_i) = r_f + (r_{ij} - r_f) + \mu_j C_i/P_i$$

which is the sum of:

The risk-free rate, r_f;

Investor j's "rent" = $r_{ij} - r_f$;

The required return of an investor with a long-term horizon = r_{ij}; and

Amortized relative trading cost of the long-term investor = μ_j Ci/Pi.

This equation is the same as that from Proposition 1, but with an extra term added, the investor rent term.

This proposition holds that there are different clienteles for securities with different degrees of illiquidity. Those that have a longer-term investment horizon will not trade as much and are less concerned with trading costs. This does not mean that they do not demand a higher return. It does mean that they do not have to have as much of an increase in return as investors who have a shorter investment horizon. This characteristic gives rise to the concave characteristic of the liquidity premium. The concave characteristic is that the liquidity premium increases, but at a decreasing rate as the degree of illiquidity increases.

Proposition 3 is a liquidity-adjusted capital asset pricing model (CAPM). In this proposition, the rate of return is not only dependent on the relative risk, but also on the illiquidity of the security and the fluctuation of illiquidity and trading costs over time.

Mathematically, this proposition may be stated as follows:

$$E(r_i) = r_f + E(c) + (r_m - c_m - r_f)\,(B_t + B_{L1} - B_{L2} - B_{L3})$$

Where:

r_f is the risk-free rate;

E(c) is expected future transaction costs;

$R_m - c_m - r_f$ is a risk premium that also incorporates expected future costs;

The betas are computed based on the correlation with the return on the market less the transaction cost in the market;

B_t is based on the relationship between the return on the security and the market;

B_{L1} is based on the relationship between the transaction cost of the security and the transaction cost of the market;

B_{L2} is based on the relationship between the return on the security and the transaction cost in the market; and

B_{L3} is based on the relationship between the transaction cost on the security and the return in the market.

It can be seen that this reformulation of the CAPM may explain the observed tendency for returns to exceed the return predicted by the CAPM as the size of a company decreases. This means that some of the observed small stock premium probably consists of a liquidity premium in addition to a risk premium. It is also probably true that some of the specific stock premium that is often added in a build-up method is probably based on a liquidity premium because the data underlying such additions are based on observed small firm effects, which are, in turn, based on liquidity as well as risk.

This proposition shows that liquidity effects are not constant over time, but vary over time. As a result, the price of assets and marketability discounts can be expected to vary over time. The behavior of observed discounts over time, such as LEAPs discounts and discounts on public companies investing in private equity, lends support to this notion. However, quantification of this effect remains difficult.

Proposition 4 is that, assuming liquidity is persistent and certain technical conditions are satisfied for a portfolio, an increase in illiquidity implies that the required return increases and contemporaneous returns are low. A high illiquidity today predicts a high illiquidity in the future. This implies a high required return, which is achieved by lowering current prices.

1.6.2 Empirical studies

In the study of liquidity, the bid-ask spread was used as one of the first indicators of liquidity. The classic proof of Proposition 2 compared the bid-ask spread of publicly traded stocks to the increase in return. It did indeed show the characteristic shape. However, numerous other indicators of liquidity are used, and there is no agreement on any one indicator being the best.

One premise in these studies is that, for smaller companies, the bid-ask spread is proportionately larger, indicating greater illiquidity. The bid-ask spread for larger stocks, on a percentage basis, is smaller, and these stocks are generally more liquid. Some of this liquidity makes its way into standard measures of risk. One way of considering risk is to look at the underlying volatility of the stock, which is sometimes measured as the change in closing price of the stock from one day to the next. For any stock, that closing price may be based either on a bid or an ask price. Thus, some of the volatility may have to do with the fact that the change from day to day is actually measuring a difference between the bid and ask prices. Because it is based on this difference, the measure of volatility also includes a measure of illiquidity. Thus, measures of rates of return based on volatility, such as CAPM beta or total beta (σ_s / σ_m), can be seen to be partially based on illiquidity. This ties into the observation that smaller companies tend to be more risky, have higher rates of return, and have greater illiquidity. For companies that do not trade, however, this effect is lessened because the closing price is stated at the average of the bid and ask prices on no trade days. It is well established that the size effect and the liquidity effect are separate, however. This is discussed in greater detail in Section 1.6.6.

The general thrust of empirical studies is that there is a liquidity effect for publicly traded common stock. Rate of return does increase as illiquidity increases. In addition, the general thrust of the studies is that the liquidity effect is concave. The rate of return increases less as the degree of illiquidity increases. However, data availability on illiquidity is less than would be desirable, and the measures of liquidity probably include errors resulting from data measurement problems.

A significant early proof of the liquidity theory is a regression of excess monthly return with bid-ask spread.[12] This regression showed a significant relation between return and bid-ask spread. The function used in the regression was a natural log of the bid-ask spread, which indicates that the result was increasing in a concave manner, that is increasing less as the amount of the bid-ask spread increased. One regression equation determined was:

$$R_j = 0.0065 + 0.0010B_j + 0.0021\ln(S_j)$$

Where:

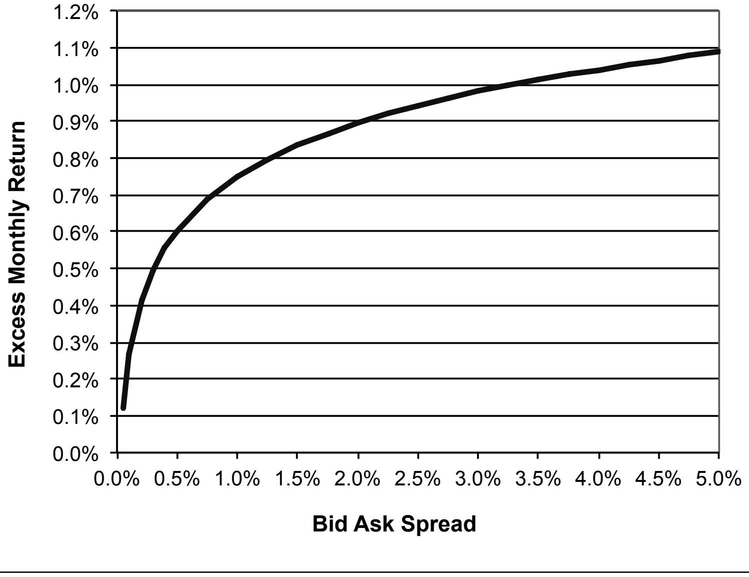
R_j = the portfolio's average monthly excess return;

B_j = previous period CAPM beta; and

S_j = the bid-ask spread from the previous year.

The equation takes the form shown in Exhibit 2 when the beta is 1.0. The bid-ask spread is a representation of illiquidity, and the excess monthly return is representative of the liquidity premium. It is the form of this graph that is important. As illiquidity increases, the liquidity premium increases at a decreasing rate. As will be discussed in later sections of the book, the rate of return is an important determinant of the DLOM, and the liquidity premium is an important part of the rate of return. It follows that the DLOM

Exhibit 2. Amihud-Mendelson BAS Return Relationship



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[12] Yakov Amihud and Haim Mendelson, "Liquidity and Stock Returns," *Financial Analysts Journal*, May-June 1986. 43-48.

will tend to follow the shape of this curve. This means that the DLOM will increase at a decreasing rate.

Unfortunately, the empirical studies have not adequately sorted out the portion of return that is related to risk and the portion that is related to liquidity in a form usable for computing DLOM. This is a difficult task because the two are intimately related. Volatility is often taken as an indication of risk. Illiquidity means an inability to sell or trade. Thus, an illiquid stock heightens the impact of volatility by lengthening the time that the holder who wants to sell is subject to the volatility. In fact, a number of the restricted stock studies that will be examined in Chapter 7 show that volatility is the factor most frequently correlated with the restricted stock discount.

The following section lists a number of studies that result in a determination of the size of the liquidity premium. It is clear that there is no consensus as to the size of the premium.

1.6.3 Size of the liquidity premium

The size of the liquidity premium is an important one because if you knew how large the liquidity premium was for a certain liquidity situation, you would know how to change what you would pay for an investment if its liquidity changed. For example, say that you had determined a price for a private company based on a publicly traded company that had a rate of return of 15%. You are valuing a privately held company, and you determine that the proper liquidity premium is 8%. Let's say the subject company produces a $1 return to you each year in perpetuity. Its value at 15% is $6.67 at the base level of liquidity. Its value at 23% (15% + 8%) is $4.35, the value of the illiquid investment.

Liquidity is being intensively studied. However, liquidity itself is very difficult to measure, and studies involving it have derived a number of different ideas to measure it. Some of these are very complex results of applying statistical analyses. It is even more difficult to measure the effects of liquidity, but this hasn't stopped academics from attempting to measure it. A large number of articles have developed estimates involving equities and various types of bonds.[13] There is no one number for the liquidity premium. It is a measurement of a difference in return between assets with two different liquidities, and the lower and higher liquidity asset or situation needs to be understood to understand where the liquidity premium applies. Although most studies find that liquidity is variable and is considered in the pricing of stock, some do not find this to be true.

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[13] The firm of Barrie Hibbert has prepared a review that provides a good summary of 35 studies that show a liquidity premium. John Hibbert, Alex Kirchner, Gavin Kretschmar, Ruosha Li, and Alexander McNeil, "Liquidity Premium Literature Review of Theoretical and Empirical Evidence Version 1.1," September 2009. www.barrhibb.com/documents/downloads/Liquidity_Premium_Literature_Review.PDF.

This research concerning the effect of liquidity on pricing is generally in two areas: (1) the effect of liquidity as an individual characteristic of the stock on the price of the stock; and (2) liquidity as a systematic or across-the-market component that fluctuates over time. The first component is termed "liquidity level" and can be diversified. The second component is called "liquidity risk" and is an undiversifiable risk factor. The bulk of this research points to the fact that liquidity affects both the individual price of a stock and the marketwide pricing of assets. However, some of the studies have not found a relationship.

A number of the studies have developed theoretical or empirical relationships that could be used for pricing an asset. The work of Pastor and Stambaugh (2003)[14] and Acharya and Pederson (2004)[15] comes to mind. However, there is such a variation in the outcomes of this literature that I hesitate to suggest any particular model. In addition, a number of these studies have resulted in estimates of the amount of the liquidity premium. However, there is substantial variation in both the amount and interpretation of these estimates, and my view is that it is too early to use a liquidity premium as a partial substitute for a risk premium. However, after reading a substantial number of these studies, I have become convinced that liquidity is an important factor affecting pricing and is a factor that should be considered, at least in a judgmental sense, in addition to risk in determining the value of a security. I have also become convinced that the rates of return that are commonly used to value securities include consideration of liquidity, but the data necessary to separate the elements of risk and liquidity do not currently exist.

A substantial number of studies have estimated a liquidity premium for various securities. Exhibit 3 shows estimates of liquidity premiums for common stock from eight different studies that are frequently cited in the literature. These estimates generally are based on the difference in return between portfolios of the most and least liquid groups of stocks, with liquidity measured in different ways in the studies. Exhibit 3 is not intended to be a complete list, but only a sampling of these studies. The wide range of the estimate of the liquidity premium shows that the jury is still out on the size of the premium.

It is also important to understand that the liquidity premium can vary substantially over time. Ben-Rephael, Kadan, and Wohl (2010)[16] focus on liquidity as a characteristic

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[14] Lubos Pastor and Robert Stambaugh, "Liquidity Risk and Expected Stock Returns," *The Journal of Political Economy,* June 2003, 111,3. 642-685.

[15] Viral Acharya and Lasse Pederson, "Asset Pricing With Liquidity Risk," *Journal of Financial Economics* 77, 2005. 375-410.

[16] Azi Ben-Rephael, Ohad Kadan, and Avi Wohl, "The Diminishing Liquidity Premium," workpaper, September 2010, papers.ssrn.com/sol3/papers.cfm?abstract_id=1099829.

Exhibit 3. Sample Indications of the Size of the Liquidity Premium		
Amihud and Mendelson (1986)	8.2% to 10.3%	
Pastor and Stambaugh (2003)	7.5%	
Acharya and Pederson (2005)	4.6%	
Sadka (2005)	6.5%	
Liu (2006)	8.2%	
Bekaert, Campbell, and Lundblad (2007)	3.2% to 7.4%	Emerging market countries
Hagströmer, Hansson, and Nilsson (2011)	1.6%	
Brennan, Chordia, and Subramanyam (2012)	6.7%	Sell side
Franzoni, Nowak, and Phalippou (2011)	2.9%	Private equity
The liquidity premiums shown are generally the difference between the most and least liquid.		

of the stock over time. They find that the sensitivity to liquidity has declined during the period from 1964 to 2008 on the NYSE and NASDAQ, but the trend is less clear on the AMEX. In other words, they find that the liquidity premium has declined during this period. However, they also observe weaker or no decline in noncommon stock (closed-end funds, REITs, and ADRs) and stocks with a price of $2 or less.

They hypothesize that the increasing use of index funds and ETFs, as well as the increase in competitive arbitrage activity, are causes of this trend. The fact that a similar usage for noncommon stocks and penny stocks is not possible tends to support this hypothesis.

The factors identified as causing the decline in the liquidity premium would also have no impact on private companies. Implications of this are important to the valuation of private companies. The work on liquidity tends to indicate that observed risk premiums and small stock premiums may be partially attributable to liquidity premiums. But if liquidity premiums for public companies have declined over time, the observed declines in risk and small stock premiums would not be applicable to private companies. In addition, this bolsters the argument that DLOMs from earlier periods would be more applicable to private companies than recent studies.

Unfortunately, these studies do not provide the data that are needed to consider liquidity for a private company. They show a premium for extremes of liquidity within the public markets. They do not show a premium that you would add to a rate of return derived from the public markets for added illiquidity in the market for private equity. Ang, Shtauber, and Tetlock (2011)[17] have prepared a working paper that compares OTCBB and Pink Sheet stocks to listed stocks and concludes that the effect of liquidity is substantially larger for the OTC stocks than for the listed stocks. Franzoni, Nowak,

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[17] Andrew Ang, Assaf Shtauber, and Paul Tetlock, "Asset Pricing in the Dark: The Cross-Section of Over-the-Counter Equities," working paper, July 2011, papers.ssrn.com/sol3/papers.cfm?abstract_id=1715463.

Exhibit 4. List of Studies Providing a Liquidity Premium

1. Yakov Amihud and Haim Mendelson, “Asset Pricing and the Bid-Ask Spread,” *Journal of Financial Economics* 17, 1986. 223-219.
2. Lubos Pastor and Robert Stambaugh, “Liquidity Risk and Expected Stock Returns,” *The Journal of Political Economy*, June 2003, 111,3. 642-685.
3. Viral Acharya and Lasse Pederson, “Asset Pricing With Liquidity Risk,” *Journal of Financial Economics* 77, 2005. 375-410.
4. Ronnie Sadka, “Momentum and Post-Earnings Announcement Drift Anomalies: The Role of Liquidity Risk,” *Journal of Financial Economics* forthcoming papers.ssrn.com/sol3/papers.cfm?abstract_id=428160.
5. Weimin Liu, “A Liquidity-Augmented Capital Asset Pricing Model,” *Journal of Financial Economics* 82, 2006. 631-671.
6. Geert Bekaert, Campbell Harvey, and Christian Lundblad, “Liquidity and Expected Returns: Lessons From Emerging Markets,” *The Review of Financial Studies*, v20, n5, 2007. 1,783-1,831.
7. Björn Hagströmer, Björn Hansson, and Birger Nilsson, “Conditional Asset Pricing With Liquidity Risk: The Illiquidity Premium,” working paper Aug. 19, 2011. www.nek.lu.se/publications/workpap/papers/WP11_24.pdf.
8. Michael J. Brennan, Tarun Chordia, Avanidhar Subrahmanyam, Qing Tong, “Sell-Order Liquidity and the Cross-Section of Expected Stock Returns,” workpaper, 2012, ssrn.com/abstract=1396328.
9. Francesco Franzoni , Eric Nowak, and Ludovic Phalippou, “Private Equity Performance and Liquidity Risk, workpaper Aug. 29, 2011, *Journal of Finance*, forthcoming, available at papers.ssrn.com/sol3/papers.cfm?abstract_id=1517044.

and Phalippou (2011)[18] have prepared a working paper that studies rates of return in the private equity markets. Using a sample of 4,403 investments from the period from 1975 to 2006 and the Pastor and Stambaugh four-factor model, liquidity is found to be a statistically significant pricing factor. At the time of the sample, the risk-free rate was 5.8% and the total cost of capital was 23.8%, resulting in a total risk premium of 18%. Of this, the liquidity premium portion was 2.9%. The alpha in the equation was insignificantly small, indicating that the four-factor formula accounted for the entire return.

In Chapter 8 of this book, I develop a relationship between volatility and the DLOM. One of the steps in developing this relationship is the estimation of a risk premium for the freely traded stock compared to the restricted stock in FMV SEC two-year era holding period data. I theorize that the liquidity premium is a function of the volatility. The FMV data used had a volatility that varied substantially, but had an average of about 75%. At this volatility, the resulting liquidity premiums for the curves I developed were between 5.7% and 10.8%, with a midpoint of 7.8%.

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[18] Francesco Franzoni, Eric Nowak, and Ludovic Phalippou, “Private Equity Performance and Liquidity Risk,” workpaper, Aug. 29, 2011, *Journal of Finance*, forthcoming, available at papers.ssrn.com/sol3/papers.cfm?abstract_id=1517044.

1.6.4 Observed determinants of illiquidity[19]

The theory discussed above provides that illiquidity results from the future trading costs of a security. These costs result from more than the direct cost incurred on a sale. Substantial academic work has developed a framework for these costs for public securities. These costs result from three categories: bid-ask spread, the price impact of buying or selling a security, and the opportunity cost of waiting to trade.

The bid-ask spread is the difference between the price at which a market maker will sell and buy a security. Bid-ask spreads are theorized to result from three factors: (1) The market maker needs to maintain an inventory; (2) the market maker incurs costs to process the transaction; and (3) the market maker experiences adverse selection, that is, there is the possibility of trading against investors with more information about the company. Empirical studies have shown relationships that correspond to these factors. Higher volume reduces the spread because the trader needs to maintain less volume. A higher security price reduces the percentage spread because the percentage processing cost per trade is less. Higher volatility increases the adverse selection and the spread because of greater risk about future information as well as several other factors.

Two factors impact the price: market illiquidity and information. Markets are not completely liquid. They cannot absorb trades in a size beyond a normal range caused by a temporary increase or decrease in price. Information effects indicate that investors must be buying or selling because they have information. This effect is smaller for larger firms, firms with high trading volume, price momentum, and high institutional holdings.

The opportunity cost of waiting to trade is the loss incurred by not executing an immediate trade. For example, a large block can be broken into smaller blocks and sold over time or smaller lots can be purchased over time. However, the price may go down, in the case of selling, or up, in the case of buying, resulting in lost profits.

The effect of these factors cannot be easily observed for private companies, but the transfer of the concepts seems straightforward. In every case, the observed factors, when correlated with bid-ask spread, price impact, and opportunity costs, will be higher for private companies. For a private company, there is very low volume, little opportunity to spread processing costs, and a substantial likelihood that an owner/manager will have greater knowledge about the company, causing a buyer to want to pay less. There are very few trades into a very thin market, raising the potential of substantial impact. The issue of time is not a willful waiting to trade, but a forced delay due to the inability to find a buyer and close a transaction.

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[19] Aswath Damodaran has written a useful paper regarding liquidity, and Section 1.6.4 is based on his summary. Aswath Damodaran, "Marketability and Value: Measuring the Illiquidity Discount," available on the Internet at pages.stern.nyu.edu/~adamodar/pdfiles/papers/liquidity.pdf.

These factors indicate that a liquidity premium and the resulting DLOM for a private company will be higher than that for a public company. The difficulty is in determining how much higher. The concave relationship for liquidity premium translates to a generally concave relationship for DLOM, meaning there is not a linear relationship of the increase as illiquidity increases. In addition, it is difficult to define liquidity and apply this definition across different classes of assets. The development of a determinative relationship between the liquidity premium and a well defined concept of liquidity would be an important find for the business valuation profession. It would be one step in the process of resolving DLOM controversies.

1.6.5 Is it double counting to deduct a DLOM when the liquidity premium is included in the rate of return?

The theory of liquidity holds that investors in securities that are harder to sell will demand a higher rate of return, which includes a liquidity premium, and thus, will pay a lower price. The theory also holds that more illiquid investments will attract a category of investors that can tolerate a longer holding period. DLOMs arise when investments are harder to sell—that is, it takes a longer time and greater transaction costs to sell them than liquid securities. There are similarities between these two concepts, longer time and lower price. This gives rise to a question of whether it is double counting to deduct a discount for lack of marketability when a liquidity premium is included in the rate of return. The likely expectations of buyers and sellers can be examined to determine whether there is double counting.

Liquidity Proposition 1 provides that it is future transaction costs that give rise to the liquidity premium. In other words, the concern of the current buyer and all future buyers with future transaction costs gives rise to higher rates of return for more illiquid securities. The future transaction costs a buyer faces include not only transaction expenses, but also costs associated with future uncertainty in the time it will take to dispose of the security. A buyer may have a desired holding period, but there is uncertainty in the actual period of time it will take to dispose of the security. This is one of the factors that causes all buyers to pay less for the security and, equivalently, to demand a higher return. The higher rates of return become one of the market factors for all participants in the market. For example, it is well known that, over a period of years, the range of expected equity rates of return in the market for private equity is in an approximate range 20% to 30%.[20] Both buyers and sellers in the market deal with this range of equity rates of return in pricing equities. The setting of this range of rates is driven primarily by the buyer's expected rate of return, which the seller accepts because it is the market rate of return. This rate of return has little to do with the specifics of the seller's difficulty in selling the security.

The seller of an illiquid security faces another hurdle, which has not necessarily made it into the elevated rates of return associated with illiquid securities. This hurdle is the immediate difficulty of selling, such as preparing the security for sale and finding a buyer. To get over this hurdle, the seller must expend additional money and time not required for a marketable security. Depending on the circumstances, preparation may include items such as preparing financial statements, preparing offering documents, and cleaning up problems such as poor accounting practices and unbooked liabilities. In addition, it will take time to find a buyer and then close on the sale. These factors add up to additional time at the front end of the sale. While this additional time is passing, the seller does not have the cash and cannot earn the opportunity cost of capital on that cash. There is foregone return. This happens despite the fact that a liquidity premium is included in the market rate of return. This is the source of the discount for lack of marketability. The DLOM could be evidenced by either the reduced return resulting from the delay of sale or by discounting the price to achieve a quicker sale. The latter option usually can't be done.

Looking at the timeline of a sale adds clarity to the situation. The seller's timeline starts when the decision to sell is made. Time passes as the security is prepared for sale and a buyer is found. The buyer's timeline starts only when the buyer is found. This is followed by closing the deal, holding the security, and then selling it later. The discount for lack of marketability is associated with the time from the seller's decision to sell to the closing of the deal. The liquidity premium is associated with the time from the buyer's decision to buy to a later closing out of the deal by the original purchaser. There is very little overlap in the time lines.

1.6.6 Is the small firm effect and liquidity double counting?

Robert Comment (2010)[21] has asserted that there is a strong empirical relation between size and liquidity and that an additional discount for illiquidity for a small firm will be redundant and result in double discounting. It is true that, on average, small firms tend to be riskier than large firms and that small firms tend to be more illiquid than large firms. However, it has also been shown that risk and illiquidity are independent effects. Thus, they should each be accounted for in a valuation. While one would not want to double count the effects, several sources point out that size and liquidity are separate factors.

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[20] See Pepperdine Private Capital Markets Project Survey Reports.

[21] Robert Comment, "Business Valuation, DLOM and *Daubert:* The Issue of Redundancy," *Business Valuation Review*, Vol. 29, No 3, 83-96.

In a section titled, "Liquidity Is Not Size," the *Ibbotson SBBI 2012 Valuation Yearbook* [22] contains a convincing discussion that the effects of liquidity impact return. It contains a table showing returns broken out by size and liquidity in each size group. In each size group, the rate of return increases as the liquidity decreases. This is particularly true in the smallest group, where the difference in return between the least and most liquid groups is 13.65%. Liquidity in these tables is measured by share turnover rates. Although the table is suggestive, it does not establish the amount of the size premium that is a liquidity premium as opposed to a risk premium.

Pratt and Abbott (2011)[23] presented a study at the October 2011 Advanced Business Valuation Conference that showed that both size and liquidity are statistically significant variables in the pricing of public company stocks, but that the relationship between them is not statistically significant. They also found that the liquidity effect is larger than the size effect.

Ang, Shtauber, and Tetlock (2011)[24] find much higher illiquidity factor returns among OTC stocks than those for listed stocks with similar market capitalizations. Thus, they find that the effect of liquidity varies among similar size firms.

Pastor and Stambaugh (2003)[25] find that "stocks that are more sensitive to aggregate liquidity have substantially higher expected returns, even after we account for exposures to the market return as well as size, value, and momentum factors." Thus, they find that sensitivity to liquidity affects return independent of size.

Brennan, Chordia, Subrahmanyam, and Tong (2012)[26] find that "sell-order illiquidity is priced more strongly in the cross-section of expected stock returns than is buy-order illiquidity. This result continues to obtain after controlling for other known determinants of expected returns such as firm size, book-to-market ratio, momentum, and share turnover." Thus, they too find that liquidity is a factor independent of size.

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[22] *Ibbotson SBBI 2012 Valuation Yearbook*, Morningstar Inc., 2010, 105.

[23] Ashok Bhardwaj Abbott and Shannon Pratt, "Size Premium and Liquidity: Substitutes or Supplements," A presentation at the 2011 Advanced Business Valuation Conference of the American Society of Appraisers.

[24] Ang, Shtauber, and Tetlock (2011).

[25] Pastor and Stambaugh (2003).

[26] Michael J. Brennan, Tarun Chordia, Avanidhar Subrahmanyam, and Qing Tong, "Sell-Order Liquidity and the Cross-Section of Expected Stock Returns," workpaper, 2012, ssrn.com/abstract=1396328.

1.7 Insight Into the IRS View on DLOM

Often the need to determine a DLOM arises in a federal tax situation. The IRS reviews these valuations, and it would be useful to understand its thinking on DLOMs. However, I am not aware that the IRS has ever published an official position regarding its stance toward DLOM. Rev. Rul. 59-60 is silent on this issue. Rev. Rul. 77-287 discusses restricted stock transactions, but provides no general guidance on the subject of DLOMs. This Revenue Ruling is presented as an appendix to Chapter 2. The IRS has issued a document titled "Discount for Lack of Marketability Job Aid for IRS Professionals, September 25, 2009." This is not an official IRS pronouncement. At the bottom of each page, it contains the following disclaimer: "This Job Aid is not Official IRS position and was prepared for reference purposes only; it may not be used or cited as authority for setting any legal position." However, this is a document the IRS prepared for use by its professionals, and it is useful in gaining an understanding of the IRS view regarding some issues relevant to DLOM.

Michael Gregory, a former IRS agent and one of the authors of the document, has stated that its purpose was to provide a consistent set of initial commentary and not to assist practitioners in preparing analyses of DLOMs. In addition, he stated that there was an IRS position that the authors of the Job Aid could not contact the sources of the various methods discussed. The methods that were discussed were those that were elevated by request to IRS business valuators in the engineering function during the 2007-to-mid-2009 time period.

This is a 112-page document, and I will not recapitulate everything that is in it. The document is freely available on the Internet at www.irs.gov/pub/irs-utl/dlom.pdf. There is, however, one item that clearly stands out and is worth pointing out. On page 74 of the Job Aid, the following quote appears: "If you are approaching the question of DLOM fresh, either as a reviewer confronted with an unreasonable taxpayer position based on invalid approaches or as a valuator charged with making your own valuation discount decisions, it is often helpful to start with a basic question as relates to DLOM. That question is: 'Under the prevailing facts and circumstances and considering the nature of the interest to be valued why is the DLOM not zero?' By enumerating the factors that would lead to a conclusion that some DLOM at all is appropriate you will be building a framework as to how substantial a discount for lack of marketability might be reasonable. This process will give you a reality check on DLOM amounts that you might ultimately derive using some of the approaches discussed in this job aid." This quote seems to indicate a philosophy of no DLOM, which is not a neutral position. It seems that a better philosophy would be to determine what the best DLOM is in the particular situation.

The bulk of this document presents items relevant to the DLOM, which it calls approaches, and discusses background information. It also presents, from the IRS point of view, areas of focus, strengths, weaknesses, prevalence in the profession, and what the courts say. The items discussed by the IRS include the following:

1. Restricted stock studies;
2. Preinitial public offering (pre-IPO) studies;
3. Restricted stock equivalent analysis;
4. Cost of flotation;
5. Mandelbaum factors, Judge Laro;
6. Long-term equity anticipation securities (LEAPS)—Robert Trout, 2003, and Ronald Seaman;
7. The Longstaff study, *Journal of Finance,* December 1995;
8. The Chaffee study;
9. Bid-ask spread method to determine DLOM;
10. Karen Hopper Wruck;
11. Hertzel and Smith;
12. Bajaj, Denis, Ferris, and Sarin;
13. Ashok B. Abbott;
14. QMDM (Christopher Mercer);
15. NICE (William Frazier);
16. NERA (David Tabak);
17. Partnership Profiles (Partnership Spectrum); and
18. Public versus private P/E ratios in acquisitions (MergerStat).

Almost all of these issues are discussed in this book. If you are considering using an approach or article that is discussed in the Job Aid, it is useful to read what the IRS has to say about it, along with the general discussion presented here. The document spends a good bit of time on restricted stock analysis and pre-IPO analysis. Valuators should be particularly aware of these criticisms so that they are properly considered in developing an analysis. This document does not plow any new ground in the area of DLOM, but does provide a good reference work on the IRS view of certain aspects of the DLOM at the time it was written, in 2009.

In addition, the Job Aid has a suggested Initial Information Document Request on pages 9 to 11 of the document. You should be aware of these and consider them as relevant in performing an analysis.